

Magician Linear Programming

Your school has contracted with a professional magician to perform at the school. The school has guaranteed an attendance of at least 1000 with total ticket receipts of at least \$4800. The tickets are \$4 for students and \$6 for non-students, of which the magician receives \$2.50 and \$4.50 respectively. How many student and non-student tickets are needed to determine a minimum amount of money for the magician?

1) Define your variables:

$$X = \text{STUDENT}$$

$$Y = \text{NON STUDENT}$$

2) The Objective Function:

$$2.5X + 4.5Y \quad \boxed{\text{MIN}}$$

3) The Constraints (find intercepts):

$$1) \quad X + Y \geq 1000 \quad (0, 1000) \quad (1000, 0)$$

$$2) \quad 4X + 6Y \geq 4800 \quad (0, 800) \quad (1200, 0)$$

5) Find any points of intersection:

$$\begin{array}{r} -4X - 4Y = -4000 \\ 4X + 6Y = 4800 \\ \hline 2Y = 800 \\ Y = 400 \end{array} \quad \begin{array}{l} X + 400 = 1000 \\ X = 600 \end{array}$$

$$(600, 400)$$

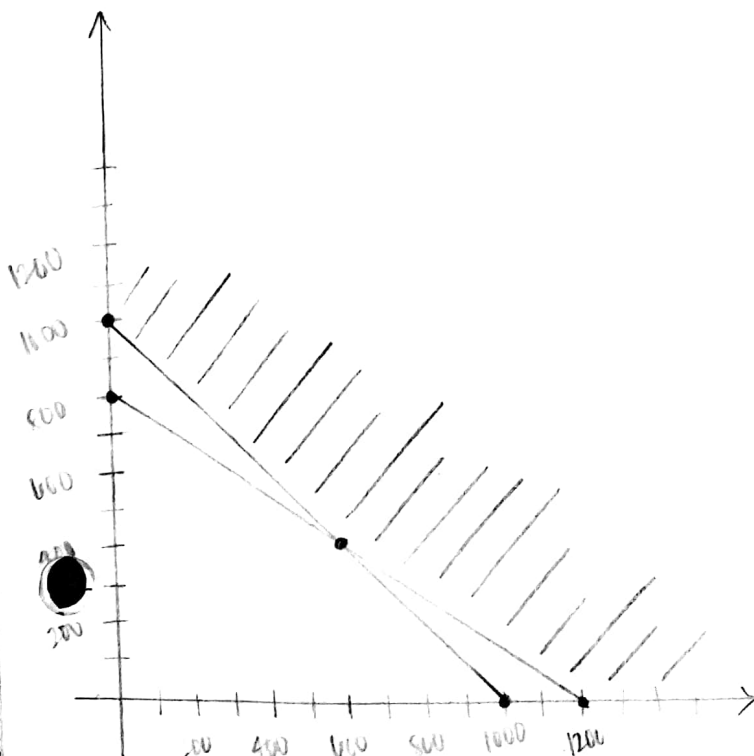
6) List the vertices of the feasible region:

$$(0, 1000)$$

$$(600, 400)$$

$$(1200, 0)$$

4) Graph the constraints (shade appropriately):



7) Plug vertices into the Objective Function to find the Max or Min:

$$2.5(0) + 4.5(1000) = 4500$$

$$2.5(600) + 4.5(400) = 3300$$

$$\boxed{2.5(1200) + 4.5(0) = 3000}$$

Freezer Linear Programming

An appliance store manager is ordering chest and upright freezers. One chest freezer costs \$250 and delivers a \$40 profit. One upright freezer costs \$400 and delivers a \$60 profit. Based on previous sales, the manager expects to sell at least 100 freezers. Total profit must be at least \$4800. Find the least number of each type of freezer the manager should order to minimize costs.

1) Define your variables:

$$X = \text{CHEST}$$

$$Y = \text{UPRIGHT}$$

2) The Objective Function:

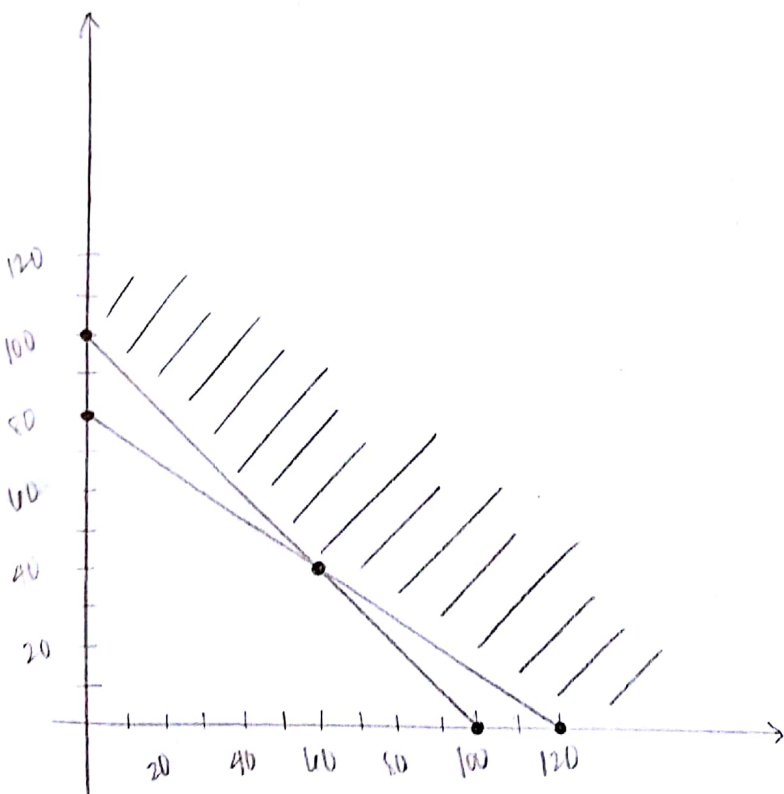
$$250X + 400Y \quad \boxed{\text{MIN}}$$

3) The Constraints (find intercepts):

$$1) \quad X + Y \geq 100 \quad \begin{matrix} (0, 100) \\ (100, 0) \end{matrix}$$

$$2) \quad 40X + 60Y \geq 4800 \quad \begin{matrix} (0, 80) \\ (120, 0) \end{matrix}$$

4) Graph the constraints (shade appropriately):



5) Find any points of intersection:

$$\begin{array}{r} -40X - 40Y = -4000 \\ 40X + 60Y = 4800 \\ \hline 20Y = 800 \\ Y = 40 \end{array}$$

$$X + 40 = 100$$

$$X = 60$$

$$(60, 40)$$

6) List the vertices of the feasible region:

$$(0, 100)$$

$$(60, 40)$$

$$(120, 0)$$

7) Plug vertices into the Objective Function to find the Max or Min:

$$250(0) + 400(100) = 40,000$$

$$250(60) + 400(40) = 31,000$$

$$\boxed{250(120) + 400(0) = 30,000}$$

the manager

Storage Space Linear Programming

An office manager is purchasing file cabinets and wants to maximize storage space. The office has 60 square feet of floor space for the cabinets and \$600 in the budget to purchase them. Cabinet A requires 3 square feet of floor space, has a storage capacity of 12 cubic feet and costs \$75. Cabinet B requires 6 square feet of floor space, has a storage capacity of 18 cubic feet, and costs \$50. How many of each cabinet should the office manager buy to maximize storage space?

1) Define your variables:

$X = \text{CABINET A}$
 $Y = \text{CABINET B}$

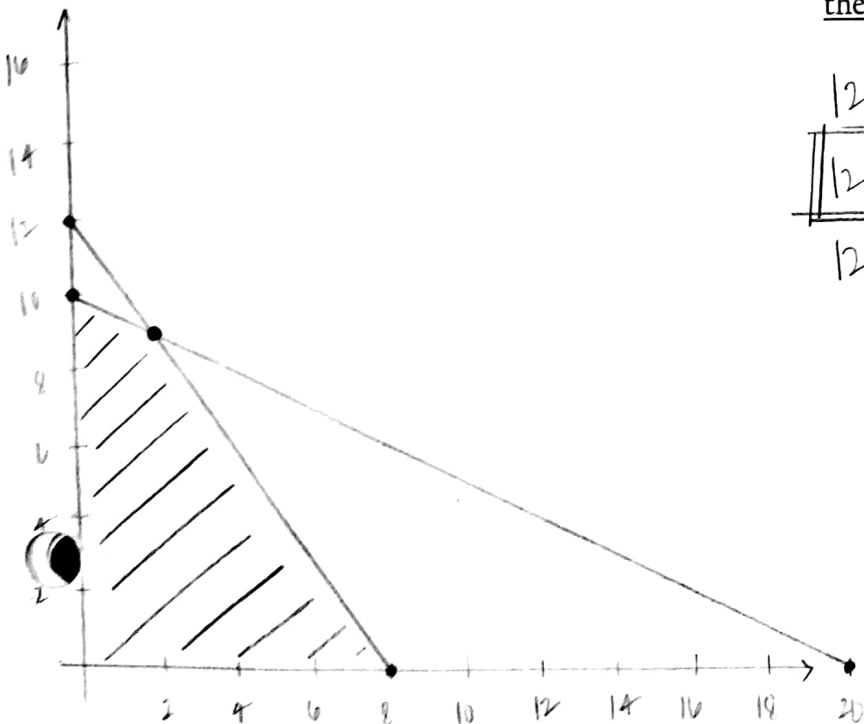
2) The Objective Function:

$12X + 18Y$ MAX

3) The Constraints (find intercepts):

1) $3X + 6Y \leq 60$ $(0, 10)$
 $(20, 0)$
2) $75X + 50Y \leq 600$ $(0, 12)$
 $(8, 0)$

4) Graph the constraints (shade appropriately):



5) Find any points of intersection:

$$\begin{array}{r} -75X - 150Y = -1500 \\ 75X + 50Y = 600 \\ \hline -100Y = -900 \\ Y = 9 \end{array}$$

$3X + 6(9) = 60$
 $3X + 54 = 6$
 $3X = 6$
 $X = 2$

$(2, 9)$

6) List the vertices of the feasible region:

- $(0, 10)$
- $(2, 9)$
- $(8, 0)$

7) Plug vertices into the Objective Function to find the Max or Min:

$12(0) + 18(10) = 180$
 $12(2) + 18(9) = 186$
 $12(8) + 18(0) = 96$

Cabinets Linear Programming

✖ You own a factory
 and skilled labor
 time, and skilled labor
 hours of machine

Your company produces cabinets using two different processes. The number of assembly hours required for each process is listed in the table. The profit from Process A is \$50 per cabinet and the profit from Process B is \$70 per cabinet. How many cabinets should you make with each process to obtain a maximum profit?

	Assembly Hours		Maximum Hours
	Process A	Process B	
Machine Time	1	2	3000
Skilled Labor	2	2	3600

1) Define your variables:

$$X = \text{PROCESS A}$$

$$Y = \text{PROCESS B}$$

2) The Objective Function:

$$50X + 70Y$$

MAX

3) The Constraints (find intercepts):

$$1) \quad X + 2Y \leq 3000 \quad (0, 1500) \quad (3000, 0)$$

$$2) \quad 2X + 2Y \leq 3600 \quad (0, 1800) \quad (1800, 0)$$

5) Find any points of intersection:

$$\begin{array}{r} -X - 2Y = -3000 \\ 2X + 2Y = 3600 \\ \hline X = 600 \end{array}$$

$$\begin{array}{r} 600 + 2Y = 3000 \\ 2Y = 2400 \\ Y = 1200 \end{array}$$

(600, 1200)

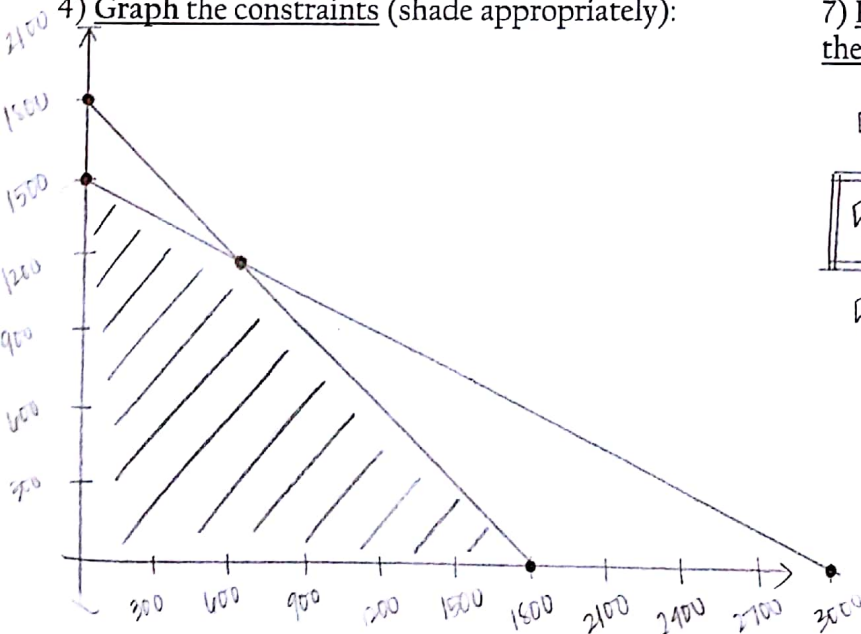
6) List the vertices of the feasible region:

$$(0, 1500)$$

$$(600, 1200)$$

$$(1800, 0)$$

4) Graph the constraints (shade appropriately):



7) Plug vertices into the Objective Function to find the Max or Min:

$$50(0) + 70(1500) = 105,000$$

$$\boxed{50(600) + 70(1200) = 114,000}$$

$$50(1800) + 70(0) = 90,000$$

Patio Sets Linear Programming

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You own a factory that makes metal patio sets using two processes. The hours of unskilled labor, machine time, and skilled labor per patio set are given in the table. You can use up to 4000 hours of unskilled labor, up to 1500 hours of machine time, and up to 2300 hours of skilled labor. Process A earns a profit of \$80 per set and Process B earns a profit of \$40 per set. How many patio sets should you make by each process to maximize profits?

	Assembly Hours		
	Process A	Process B	
Unskilled Labor	10	1	4000
Machine Time	1	3	1500
Skilled Labor	5	2	2300

1) Define your variables:

$$X = \text{PROCESS A}$$

$$Y = \text{PROCESS B}$$

2) The Objective Function:

$$80X + 40Y \quad \boxed{\text{MAX}}$$

3) The Constraints (find intercepts):

$$1) \quad 10X + Y \leq 4000 \quad (0, 4000) \quad (400, 0)$$

$$2) \quad X + 3Y \leq 1500 \quad (0, 500) \quad (1500, 0)$$

$$3) \quad 5X + 2Y \leq 2300 \quad (0, 1150) \quad (460, 0)$$

4) Graph the constraints (shade appropriately):

on separate paper.

5) Find any points of intersection:

6) List the vertices of the feasible region:

$$(0, 500)$$

$$(300, 400)$$

$$(380, 200)$$

$$(400, 0)$$

7) Plug vertices into the Objective Function to find the Max or Min:

$$80(0) + 40(500) = 20\,000$$

$$\boxed{80(300) + 40(400) = 40\,000}$$

$$80(380) + 40(200) = 32\,400$$

$$80(400) + 40(0) = 32\,000$$

nd 3 hours

4000
①

3500

3000

2500

2000

1500

1250

1000

500
②

(0, 500)

Intersection 2 + 3

$$-5X - 15Y = -7500$$

$$5X + 2Y = 2300$$

$$-13Y = -5200$$

$$Y = 400$$

$$X + 3(400) = 1500$$

$$X + 1200 = 1500$$

$$X = 300$$

(300, 400)

Intersection 1 + 3

$$-20X - 2Y = -8000$$

$$5X + 2Y = 2300$$

$$-15X = -5700$$

$$X = 380$$

$$10(380) + Y = 4000$$

$$3800 + Y = 4000$$

$$Y = 200$$

(380, 200)

③

②

(300, 400)

(380, 200)

(400, 0)

100

200

300

400

500

600

Income Linear Programming

You are stenciling wooden boxes to sell at a craft fair. It takes you 2 hours to stencil a small box and 3 hours to stencil a large box. You make a profit of \$10 for a small box and \$20 for a large box. You have no more than 30 hours available to stencil and want at least 12 boxes to sell. How many of each size box should you stencil to maximize your profit?

VARIABLES

X = SMALL

Y = LARGE

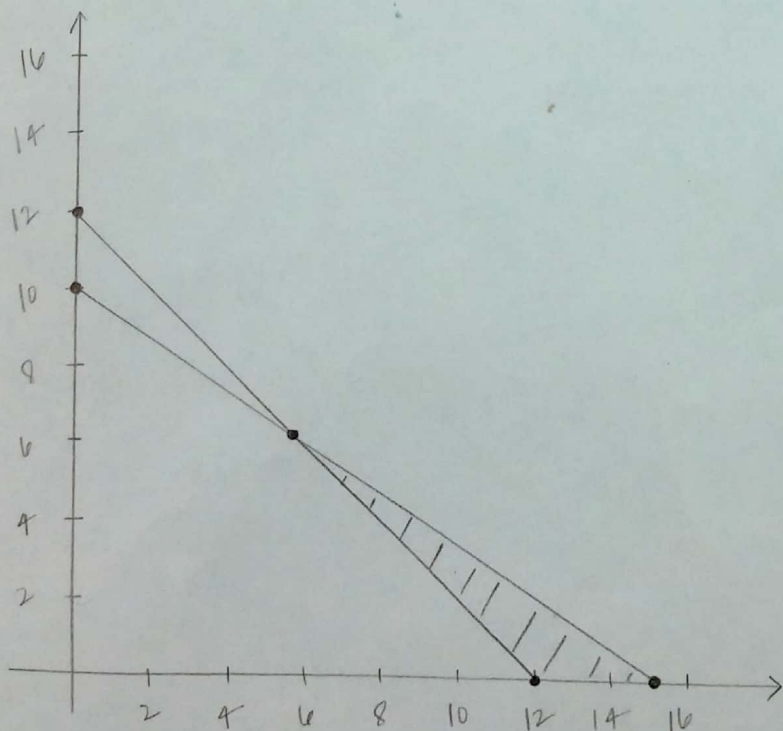
OBJECTIVE

$$10X + 20Y \quad \boxed{\text{MAX}}$$

CONSTRAINTS

$$\begin{array}{l} 1) \ 2X + 3Y \leq 30 \\ 2) \ X + Y \geq 12 \end{array} \quad \begin{array}{l} (0, 10) \\ (15, 0) \\ (0, 12) \\ (12, 0) \end{array}$$

GRAPH



FIND INTERSECTION

$$\begin{array}{r} 2X + 3Y = 30 \\ -2X - 2Y = -24 \\ \hline \end{array}$$

$$Y = 6$$

$$(6, 6)$$

$$X + Y = 12$$

$$X = 6$$

VERTICES

$$(6, 6)$$

$$(12, 0)$$

$$(15, 0)$$

ANSWER

$$\boxed{10(6) + 20(6) = 180}$$

$$10(12) + 20(0) = 120$$

$$10(15) + 20(0) = 150$$